一、選擇題

1. (5%) Professor X lectures the course "Probability" in a university. He arrives on time for the lecture 80% of the time, and when he is late, the arrival time delay is uniformly distributed from 0 to 20 minutes. Irrespective of the arrival time, Professor X ends the lecture on time, which is 60 minutes after the scheduled starting time. Assume that all students in the university can wait for at most 10 minutes for a lecture to begin such that they leave the class 10 minutes after the scheduled starting time without the appearance of Professor X. Let Y be the expected duration of time that students observe a lecture, then

   (A) $0 < Y \leq 50$.

   (B) $50 < Y \leq 52$.

   (C) $52 < Y \leq 54$.

   (D) $54 < Y \leq 56$.

   (E) $Y > 56$.

2. (5%) You are invited to a tournament of board games in which you play game after game until you lose one. Assume that for each game the probability that you win, lose, or get in a tie is equal, and the outcome of each game is independent of the outcome of every other game. For each game, you earn 2 points for each win, 1 point for each tie, and 0 points for each loss. Let Z equal the total number of points that you earn in the tournament, then

   (A) $0 < E[Z] \leq 2$.

   (B) $2 < E[Z] \leq 4$.

   (C) $4 < E[Z] \leq 6$.

   (D) $6 < E[Z] \leq 8$.

   (E) $E[Z] > 8$.

3. (5%) (*Continued from Problem 2 on the tournament of board games.*) Let $\text{Var}[Z]$ be the variance of Z in Problem 2, then

   (A) $0 < \text{Var}[Z] \leq 2$.

   (B) $2 < \text{Var}[Z] \leq 4$.

   (C) $4 < \text{Var}[Z] \leq 6$.

   (D) $6 < \text{Var}[Z] \leq 8$.

   (E) $\text{Var}[Z] > 8$.

4. (5%) Of the following random variables with the same expected value q, where $0 < q < 1$, which has the largest variance?

   (A) Poisson random variable.

   (B) Bernoulli random variable.

   (C) Continuous uniform random variable (starting from 0).

   (D) Exponential random variable.

   (E) *Unable to determine with the given statement.*

5. (5%) Of the following mathematicians with names associated with the theory of probability, whose birth date is the closest to our days?

   (A) Andrey Kolmogorov (for Axioms of Probability).

   (B) Pafnuty Chebyshev (for Chebyshev Inequality).

   (C) Andrey Markov (for Markov Inequality).

   (D) Herman Chernoff (for Chernoff Bound).

   (E) Agner Erlang (for Erlang Distribution).

6. (5%) (**Multiple Choices**) Choose the wrong statement(s) about a Gaussian random vector $\mathbf{X} = [X_1\ X_2\ \cdots\ X_n]'$ with covariance matrix $\mathbf{C_X}$:

   (A) Any linear combination of its components $\{X_1, X_2, \ldots, X_n\}$ follows the normal distribution.

   (B) Any subset of its components $\{X_1, X_2, \ldots, X_n\}$ is jointly Gaussian.

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(C) $\mathbf{X}$ has independent components if and only if $\mathbf{C_X}$ is diagonal.

(D) There always exists a matrix $\mathbf{A}$ such that $\mathbf{C_X} = \mathbf{AA}'$, where $\mathbf{A}'$ is the transpose of $\mathbf{A}$.

(E) None of the above.

二、非選擇題

1. (10%) Assume that you arrive at a bus stop at time 0. At the end of each minute, a bus arrives with probability p, and no bus arrives with probability 1-p. Whenever a bus arrives, you board the bus with probability q. Let T be the amount of time (in minutes) you wait at a bus stop, and N be the number of buses that arrive while you are waiting (including the one that you finally board).

   (a) Find the joint PMF $P_{N,T}(n,t)$.

   (b) Find the conditional PMF $P_{N|T}(n|t)$.

2. (10%) Given the set $\{U_1,\ldots,U_n\}$ of *i.i.d.* continuous uniform $(0,T)$ random variables, define

$$X_k = \text{small}_k(U_1,\ldots,U_n)$$

as the k^{th} smallest element of the set. Find the probability distribution function $f_{X_1,\ldots,X_n}(x_1,\ldots,x_n)$.

3. Find the general solution of the following differential equations: (24 scores)

(a) $y^{(3)}(x)+2y''(x)+y'(x)=1$

(b) $\dfrac{dy(x)}{dx}=\left[y(x)+x\right]^3-1$

(c) $\begin{cases}\dfrac{d}{dt}x(t)=y(t)\\ \dfrac{d}{dt}y(t)=z(t)\\ \dfrac{d}{dt}z(t)=x(t)\end{cases}$

4. Find the inverse Laplace transform of (9 scores)

$$\frac{2s^2+3s+4}{s^2+2s+3}$$

5. Find the Fourier series of (8 scores)

$$f(x)=\begin{cases}0.5, & 0<x<3\\ -0.5, & 3<x<6\end{cases}\qquad f(x)=f(x+6)$$

6. Solve the following partial differential equation (9 scores)

$$\frac{\partial^2}{\partial x^2}u(x,y)=5\frac{\partial}{\partial x}\frac{\partial}{\partial y}u(x,y)$$

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