國立臺灣大學 104 學年度碩士班招生考試試題

※ 注意：請於試卷內之「非選擇題作答區」依序作答，並應註明作答之大題及小題題號。

(1) (20 pts) Suppose that f is a twice-differentiable real-valued function on the real line such that $|f(x)| \leq 1$ and $|f''(x)| \leq 1$ for all x. Find, with proof, a constant b such that $|f'(x)| < b$ for all x.

(2) (20 pts) (a) (12 pts) Evaluate the integral

$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx.$$

(b) (8 pts) Prove that $0 < \frac{22}{7} - \pi < \frac{1}{256}$.

(3) (20 pts) (a) (15 points) Draw the graphs of $y = x^4 + ax^2$ for $a \geq 0$ and $a < 0$. In your graph, it should address the symmetry of the graph, the number of local minimal, and the number of local maximum.

(b) (5 points) For which real numbers c is there a straight line that intersects the curve

$$y = x^4 + 9x^3 + cx^2 + 9x + 4$$

in four distinct points?

(4) (10 pts) Find all real-valued continuously differentiable functions f on the real lines such that for all x

$$(f(x))^2 = \int_0^x [(f(t))^2 + (f'(t))^2] dt + 1990.$$

(5) (15 pts) Evaluate $\int\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$, where $\mathbf{F}(x) = (xy, y, z)$ and S is the surface described by $x^2 + y^2 + z^2 = 2$ and $z \geq 1$. (S is oriented with the upward unit normal.)

(6) (15 pts) Let S be the surface of the tetrahedron whose vertices are $(1,0,0)$, $(0,1,0)$, $(0,0,2)$ and the origin. Evaluate $\int\int_{\mathrm{S}} f d\mathrm{S}$ where $f(x,y,z) = xz$.

試題隨卷繳回