國立臺灣大學 104 學年度碩士班招生考試試題

※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

For Questions 1 to 10, select a correct answer for each question and mark the letter (a), (b), (c), or (d) on your answer card.

1. (5%) For speeds between 40 and 65 kilometers per hour, a truck gets $480/x$ kilometers for per liter of diesel gasoline when driven at a constant speed of x kilometers per hour. Diesel gasoline costs \$2.23 per liter, and the driver is paid \$15.10 per hour. What is the most economical constant speed between 40 and 65 kilometers per hour at which to drive the truck?
   (a) 52.5 kilometers per hour
   (b) 57 kilometers per hour
   (c) 65 kilometers per hour
   (d) None of the above

2. (5%) What is the value of $\int_1^2 x^3 \ln(\sqrt{x})dx$?
   (a) 0.713
   (b) 0.865
   (c) 0.918
   (d) None of the above

3. (5%) Which can be the solution form of $A(t)$ if
$$A'(t) = A^2(t) + A(t) - 1, \text{ and } A(0) = 0.$$
   (a) $A(t) = a + be^{ct} + d$
   (b) $A(t) = \exp(a + be^{ct}) + d$
   (c) $A(t) = (a + be^{ct})^{-1} + d$
   (d) None of the above

4. (5%) Consider a cash-or-nothing call option which pays the option holder an amount K at the maturity T only if the asset price at T is larger than K. The pricing formula for this option is as follows.
$$c = Ke^{-rT}N(d),$$
where
$$d = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$
and S is the current asset price, r is a constant risk-free interest rate, σ is the volatility of the asset price, and $N(\cdot)$ is the cumulative distribution function of the standard normal distribution defined as
$$N(d) = \int_{-\infty}^{d} n(x)dx = \int_{-\infty}^{d} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx.$$

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Derive the Delta (i.e., $\frac{\partial c}{\partial S}$) of this option,.

(a) $\frac{1}{\sqrt{2\pi}\sigma\sqrt{T}}e^{-\frac{(d+\sigma\sqrt{T})^2}{2}}$

(b) $\frac{K}{\sqrt{2\pi}S\sigma\sqrt{T}}e^{-\frac{d^2}{2}}$

(c) 0

(d) None of the above

5. (5%) For the pricing formula of the cash-or-nothing call option in Question 4, derive the vega (i.e., $\frac{\partial c}{\partial \sigma}$) of this option.

(a) $\frac{K}{\sqrt{2\pi}}e^{-rT+\frac{d^2}{2}}(\sqrt{T}+\frac{d}{\sigma})$

(b) $\frac{K}{\sqrt{2\pi}}e^{-rT+\frac{d^2}{2}}(\sqrt{T}-\frac{d}{\sigma})$

(c) $\frac{-K}{\sqrt{2\pi}}e^{-rT-\frac{d^2}{2}}(\sqrt{T}+\frac{d}{\sigma})$

(d) None of the above

6. (5%) What is the value of $\lim_{x\to 0}\left(\frac{1}{1-\cos x}-\frac{2}{x^2}\right)$?

(a) $-\infty$
(b) 1/3
(c) 1/6
(d) ∞

7. (5%) Determine whether the following statement is true or false.

"Suppose a_n is positive for all n. If $\sum_{i=1}^{\infty} a_n$ converges, then $\sum_{i=1}^{\infty}\sqrt{a_n a_{n+1}}$ also converges."

(a) False
(b) True
(c) Uncertain (need more information)
(d) None of the above

8. (5%) A community is laid out as a rectangular grid in relation to two main streets that intersect at the city center. Each point in the community has coordinates (x, y) in this grid, for $-10 \le x \le 10$, $-8 \le y \le 8$ with x and y measured in

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kilometers. Suppose the value of the land located at the point (x,y) is V thousand dollars, where

$$V(x,y) = (250 + 17x)e^{-0.01x-0.05y}.$$

Estimate the value of the block of land occupying the rectangular region $1 \le x \le 3,\ 0 \le y \le 2$.

(a) 759 thousand dollars
(b) 859 thousand dollars
(c) 959 thousand dollars
(d) None of the above

9. If a positive variable λ follows a gamma distribution, i.e., λ~Gamma(α,β), where α and β are positive real numbers. Consequently, the probability density function for λ is as follows.

$$f(\lambda) = \beta^{\alpha}\frac{1}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda},$$

where $\Gamma(Z)$ is the gamma function and defined as

$$\Gamma(Z) = \int_0^{\infty} e^{-t}t^{Z-1}dt,$$

if Z is a complex number with a positive real part. What is the var(λ) given $\alpha = 1$ and $\beta = 2$? (5%)

(a) 1/4
(b) 1/2
(c) 1
(d) None of the above

10. (5%) The death rate and birth rate of many animal and plant species fluctuate periodically with the seasons. The population $P(t)$ of such a species at time t changes at a rate that may be modeled by a differential equation of the form

$$\frac{dP}{dt} = (1 + \cos 2\pi t)P.$$

Given the initial population $P(0)$ to be 1000, what is the population $P(1.5)$?

(a) 1482
(b) 2482
(c) 3482
(d) None of the above

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For Questions 11 to 15, show your calculations/proof **in detail** on the answer sheet.

11. (10%) Find dy/dx given that $y^3 + y^2 - 5y - x^2 = -4$.

12. (10%) Evaluate

$$\int_0^1 \arcsin(x)\, dx.$$

13. (10%) Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{2^k x^k}{k}$.

14. (10%) Find the surface area of the portion of the plane $x + y + z = 1$ that lies in the first octant (where $x \geq 0, y \geq 0, z \geq 0$).

15. (10%) Suppose X is a metric space and K is a subset of X. Describe and explain the definition of K being a compact set.

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