※ 注意：請於試卷上「非選擇題作答區」內依序作答，並應註明作答之大題及其題號。

There are four problems 1 ~ 4 in total; some problems contain sub-problems, indexed by (a), (b), etc.

1. (a) [10%] Let $u(x), v(x)$ be positive and differentiable on an open interval. Find the derivative of the function

$$[u(x)]^{v(x)}.$$

   (b) [15%] Let T be the surface in $\mathbb{R}^3$ given by the equation

$$(\sqrt{x^2+y^2}-2)^2+z^2=1.$$

   Compute the surface integral

$$\int_T |z|\,\mathrm{d}S$$

   where $\mathrm{d}S$ denotes the element of surface.

2. (a) [15%] Let $f(x)=\sum_{k=0}^{\infty} a_k x^k$ be a power series ($a_k \in \mathbb{R}$). Suppose that $f(x)$ converges at c where $c>0$. Show that $f(x)$ converges at any x with $|x|<c$.

   (b) [10%] Give an explicit example of a power series which converges exactly on the interval $(-1,1]$. You need to justify your answer.

3. Consider the improper integral

$$G(s)=\int_0^{\infty} e^{-x}x^{s-1}\,\mathrm{d}x.$$

   (a) [15%] Show that the integral $G(s)$ exists and defines a continuous function for $s>0$.

   (b) [10%] Evaluate $G(\frac{1}{2})$.

4. [25%] Let $F(x,y)$ and $G(x,y)$ be differentiable on an open domain S in $\mathbb{R}^2$. Suppose under the condition $G(x,y)=0$ that $F(x,y)$ reaches its maximal at the point $(a,b)\in S$. Show that if $G_x(a,b)\neq 0$, there exists a real constant λ such that

$$\begin{aligned} F_x(a,b) &= \lambda\cdot G_x(a,b)\\ F_y(a,b) &= \lambda\cdot G_y(a,b). \end{aligned}$$

試題隨卷繳回